Problem 29.8

A charge moving with velocity $\vec{v} = \left(2\hat{i} - 4\hat{j} + \hat{k}\right)(\text{m/s})$ is in a region where there exists a magnetic field of $\vec{B} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)(\text{T})$.

There are a number of ways to evaluate this cross product.

1.) Use $|\vec{v}| = \left(v_x^2 + v_y^2 + v_y^2\right)^{1/2}$ to get the magnitude of the velocity, do the same with the magnetic field, then determine the angle between the two vectors (not an altogether easy feat as we are talking about three-dimensional vectors and a spherical-polar problem). Then use the right-hand rule to determine direction.

2.) Do individual mini cross-products. That is, execute the series of cross products

$$\vec{v}\text{x}\vec{B} = \left(v_x\hat{i} + v_y\hat{j} + v_y\hat{k}\right)\text{x}\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(v_x\hat{i}\right)\text{x}\left(B_x\hat{i}\right) + \left(v_x\hat{i}\right)\text{x}\left(B_y\hat{j}\right) + \left(v_x\hat{i}\right)\text{x}\left(B_z\hat{k}\right) + \ldots$$

until you've executed all nine mini cross products. The sum of these in unit vector notation will give the net force.

A charge moving with velocity $\vec{v} = \left(2\hat{i} - 4\hat{j} + \hat{k}\right)(\text{m/s})$ is in a region where there exists a magnetic field of $\vec{B} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)(\text{T})$.

3.) The most reasonable is to evaluate the cross product as done when a unit-vector notation is used. That is, evaluate the matrix shown below (note that the unit vectors are arrayed in the first row, the velocity components in the second row and the magnetic field components in the third row):

$$\vec{F} = q\vec{v}\text{x}\vec{B}$$
$$= q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

$\vec{v} = \left(2\hat{i} - 4\hat{j} + \hat{k}\right)(\text{m/s})$ $\quad$ $\vec{B} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)(\text{T})$.

That process yields:

$$\vec{F} = q\vec{v}\text{x}\vec{B}$$
$$= q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 2 & -1 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 2 & -4 \\ 1 & 2 \end{matrix}$$
$$= \left(1.6\text{x}10^{-19}\ \text{C}\right)\left[\left(\hat{i}\right)\left[(-4)(-1)-(1)(2)\right] + \left(\hat{j}\right)\left[(1)(1)-(2)(-1)\right] + \left(\hat{k}\right)\left[(2)(2)-(-4)(1)\right]\right]$$
$$= \left(1.6\text{x}10^{-19}\ \text{C}\right)\left[\left(2\hat{i}\right) + \left(3\hat{j}\right) + \left(8\hat{k}\right)\right]$$
$$= \left[\left(3.2\text{x}10^{-19}\right)\hat{i} + \left(4.9\text{x}10^{-19}\right)\hat{j} + \left(12.8\text{x}10^{-19}\right)\hat{k}\right](\text{N})$$

The magnitude of the vector is:

$$\left|\vec{F}\right| = \left[\left(3.2\text{x}10^{-19}\right)^2 + \left(4.9\text{x}10^{-19}\right)^2 + \left(12.8\text{x}10^{-19}\right)^2\right]^{1/2}$$
$$= 1.4\text{x}10^{-18}\ \text{N}$$